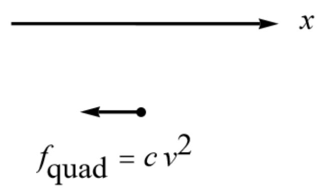
Problem 2.26

A typical value for the coefficient of quadratic air resistance on a cyclist is around $c = 0.20\ \text{N}/(\text{m/s})^2$. Assuming that the total mass (cyclist plus cycle) is $m = 80$ kg and that at $t = 0$ the cyclist has an initial speed $v_o = 20$ m/s (about 45 mi/h) and starts to coast to a stop under the influence of air resistance, find the characteristic time $\tau = m/cv_o$. How long will it take him to slow to 15 m/s? What about 10 m/s? And 5 m/s? (Below about 5 m/s, it is certainly not reasonable to ignore friction, so there is no point pursuing this calculation to lower speeds.)

Solution

Draw a free-body diagram for a cyclist travelling to the right in a medium with quadratic air resistance.



Apply Newton's second law in the x-direction.

$$\sum F_x = ma_x$$

Let $v_x = v$ to simplify the notation.

$$-cv^2 = m\frac{dv}{dt}$$

Solve this differential equation for v by separating variables.

$$-\frac{c}{m}\,dt = \frac{dv}{v^2}$$

Integrate both sides definitely, assuming that at $t = 0$ the velocity is v_o.

$$\int_0^t -\frac{c}{m}\,dt' = \int_{v_o}^{v} \frac{dv'}{v'^2}$$

$$-\frac{c}{m}t'\Big|_0^t = -\frac{1}{v'}\Big|_{v_o}^{v}$$

$$-\frac{c}{m}(t-0) = -\left(\frac{1}{v} - \frac{1}{v_o}\right)$$

$$t = \frac{m}{c}\left(\frac{1}{v} - \frac{1}{v_o}\right)$$

With $c = 0.20\ \text{N}/(\text{m/s})^2$, $m = 80$ kg, and $v_\text{o} = 20$ m/s, this equation becomes

$$t = \frac{80}{0.20}\left(\frac{1}{v} - \frac{1}{20}\right).$$

Therefore, it will take the cyclist

$$t = \frac{80}{0.20}\left(\frac{1}{15} - \frac{1}{20}\right) \approx 6.67 \text{ seconds}$$

to slow to 15 meters per second,

$$t = \frac{80}{0.20}\left(\frac{1}{10} - \frac{1}{20}\right) = 20 \text{ seconds}$$

to slow to 10 meters per second, and

$$t = \frac{80}{0.20}\left(\frac{1}{5} - \frac{1}{20}\right) = 60 \text{ seconds}$$

to slow to 5 meters per second. The characteristic time is

$$\tau = \frac{m}{cv_\text{o}} = \frac{80 \text{ kg}}{\left(0.20\ \frac{\text{N}\cdot\text{s}^2}{\text{m}^2}\right)\left(20\ \frac{\text{m}}{\text{s}}\right)} = 20 \text{ seconds.}$$

Below is a graph (in SI units) of the cyclist's velocity as a function of time.